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$f(x) = \frac{4-x}{\ln(4-x)}$; $D =]-\infty ; 4[\setminus \{3\}$

1.1 $x \to -\infty$: $f(x) = \frac{4-x}{\ln(4-x)} \to$ "$\frac{\infty}{\infty}$" $\xrightarrow{\text{l.H}} \frac{-1}{\frac{1}{4-x}\cdot(-1)} = 4-x \to \infty$

$x \to 3^-$: $f(x) = \frac{4-x}{\ln(4-x)} \to$ "$\frac{1^+}{\ln(1^+)}$" $\to$ "$\frac{1}{0^+}$" $= +\infty$

$x \to 3^+$: $f(x) \to$ "$\frac{1}{\ln(1^-)}$" $\to$ "$\frac{1}{0^-}$" $= -\infty$

$x \to 4^-$: $f(x) \to$ "$\frac{0}{\ln(0^+)}$" $\xrightarrow[\text{s.o}]{\text{l.H}} 4-x = 0$

1.2 $f'(x) = \frac{\ln(4-x)\cdot(-1) - (4-x)\cdot\frac{1}{(4-x)}\cdot(-1)}{[\ln(4-x)]^2} = \frac{1-\ln(4-x)}{[\ln(4-x)]^2}$

$x \to 4$: $f'(x) = \frac{1-\ln(4-x)}{[\ln(4-x)]^2} \to$ "$\frac{1+\infty}{(-\infty)^2}$"

$\xrightarrow{\text{l.H}} \frac{-\frac{1}{4-x}\cdot(-1)}{2\ln(4-x)\cdot\frac{1}{4-x}\cdot(-1)} = \frac{-1}{2\ln(4-x)} \to$ "$\frac{-1}{-\infty}$" $= 0^+$

1.3 $f'(x) = \frac{1-\ln(4-x)}{[\ln(4-x)]^2} = 0 \Leftrightarrow \ln(4-x) = 1 \Leftrightarrow 4-x = e$; $N(x) > 0$

$\Leftrightarrow x_E = 4-e$ $(\approx 1{,}3)$ m. VZW. da $4-x$ VZW hat

Art d. Extremums:

- Wegen $f(x) \to \infty$ für $x \to -\infty$, $f(x) \to \infty$ für $x \to 3^-$ } TIP bei $x_E = 4-e$
- $f'(0) \approx -0{,}20 < 0$, $f'(2) \approx 0{,}64 > 0$ } VZ f': − 0 + bei x_E; f: smf TIP sms ; $f(4-e) = \frac{e}{\ln(e)} = e$; TIP $(4-e \mid e)$
- $f''(x) = \ldots$ zu kompliziert

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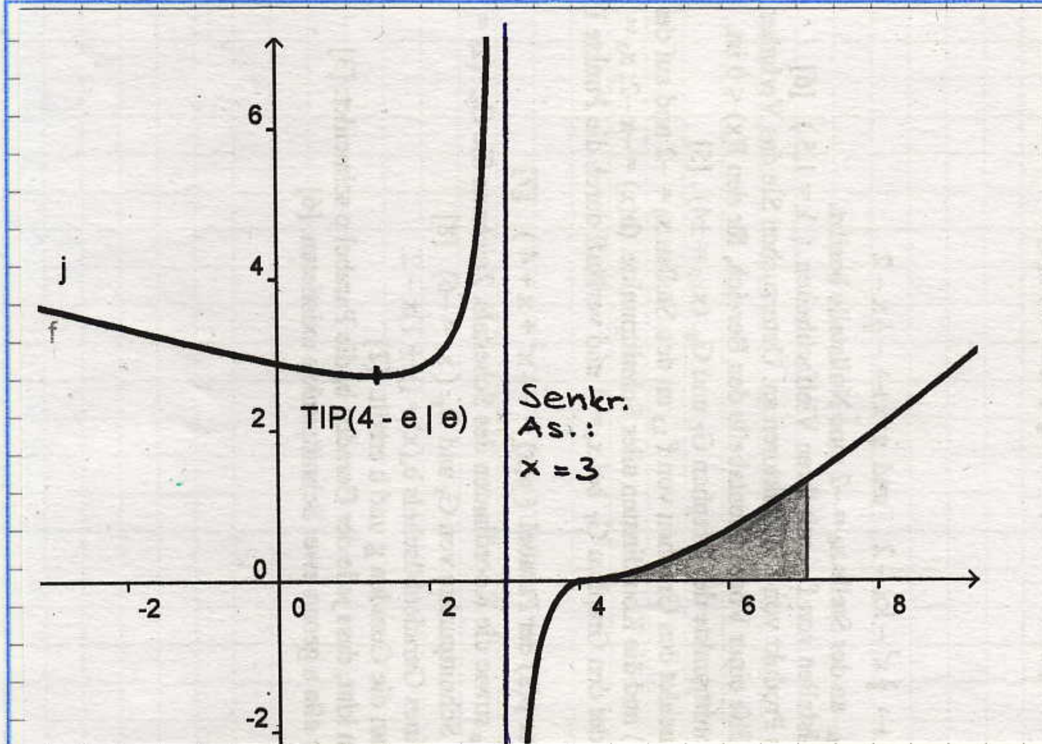

TIP für GeoGebra

$t(x) = \frac{x^2 - 8x + 16}{x}$; $f(x) = \frac{4-x}{\ln(4-x)}$

$g(x) = \text{Wenn}[x < 4 \mid f(x) \mid t(x)]$

2.1 $\lim\limits_{x \to 4^+} g_a(x) = \frac{16 - 32 + a^2}{4} \stackrel{!}{=} 0 \Leftrightarrow a^2 = 16 \Rightarrow \underline{a_{1/2} = \pm 4}$

2.2 $g_4(x) = x - 8 + \frac{16}{x} = x - 8 + 16x^{-1}$

$g_4'(x) = 1 - 16x^{-2}$; $g_4'(4) = 1 - \frac{16}{16} = 0$ $\Big\}\ x \geq 4$

aus 1.2 : $g'(x) = f'(x) \to 0$ f. $x \to 4^-$ $\Big\}$ $\underline{\text{gleich} \Rightarrow \text{d. bar}}$

2.3 Tangente : $\underline{y = 0}$; Normale : $\underline{x = 4}$

$g(x) = x - 8 + 16 \cdot \frac{1}{x} \Rightarrow G(x) = \frac{1}{2}x^2 - 8x + 16 \ln(x)$

$A = \int_4^7 g(x)\,dx = G(7) - G(4)$

$= \frac{1}{2} \cdot 49 - 8 \cdot 7 + 16 \ln(7) - \left[\frac{1}{2} \cdot 16 - 8 \cdot 4 + 16 \ln(4)\right]$

$= -\frac{15}{2} + 16 \ln(7) - 16 \ln(4)$

$= \underline{-\frac{15}{2} + 16 \ln\left(\frac{7}{4}\right)}$ $(\approx 1{,}45)$